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A method to study "chemical" fluctuations in nucleus–nucleus collisions

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Abstract. A method to study event–by–event fluctuations of the "chemical" (particle type) composition of the final state of high energy collisions is proposed.

1 Introduction

Recent data on hadron production in central nucleus–nucleus (A+A) collisions at the CERN SPS are compatible with the hypothesis that a Quark Gluon Plasma [1] is created in the early stage of the interaction [2].

This interpretation however requires the assumption that the produced matter is close to thermal and chemical equilibrium. Thus it is important to measure the level of equilibration reached in nuclear collisions.

The method to study event–by–event fluctuations of kinematical variables ("thermal" fluctuations, TF–method) was proposed in 1992 [3] and recently used for the analysis of central Pb+Pb collisions at 158 A·GeV by the NA49 Collaboration [4]. First results indicate that the analysis of event–by–event fluctuations adds crucial information concerning the dynamics of A+A collisions and in particular allows to reject purely non–equilibrium approches, like initial state scattering models [5]. The amount of "thermal" fluctuations was recently calculated for matter in equilibrium [6].

In this paper we propose a method to study event–by–event fluctuations of the particle composition of the final state of high energy collisions ("chemical" fluctuations, CF–method). We expect that this method will allow to determine whether or not chemical equilibrium is reached in high energy collisions. As it is based on the TF–method, we start (Sect. 1) from a brief description of the basic idea and the formalism of the TF–method [3]. In Sect. 3 we introduce the CF–method. A simple numerical example is presented in Sect. 4.

2 Description of the TF–method

Let us suppose that A+A collisions can be modeled as a sum of independent nucleon–nucleon (N+N) interactions. In this case event–by-event fluctuations in A+A collisions are given by a superposition of the fluctuations present in N+N interactions. Additional fluctuations are introduced when the number of superimposed N+N interactions varies from event to event due to e.g. variation in the collision geometry.

A statistical method which allows to remove the influence of trivial geometrical fluctuations and the effect of averaging over many particle sources was introduced in [3]. It was proposed to quantify the fluctuations by the so–called Φ variable. As an example we consider here a construction of Φ for the case of transverse momentum fluctuations.

For every particle i one defines:

$$z_i = p_{T_i} - \overline{p_T},$$

where $\overline{p_T}$ is the mean transverse momentum calculated for all particles from all events (the inclusive mean). Using z_i one calculates for every event

$$Z = \sum_{i=1}^{N} z_i,$$

where N is the number of analyzed particles in the event. The fluctuation measure[1], Φ_{p_T}, is then defined as:

$$\Phi_{p_T} = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}, \qquad (1)$$

where $\langle N \rangle$ and $\langle Z^2 \rangle$ are averages (of event–by–event observables) over all events and the second term in the r.h.s. is the square root of the second moment of the inclusive z distribution.

By construction [3], the Φ_{p_T} value for A+A collisions is equal to the Φ_{p_T} value for N+N interactions in the case in which A+A collisions are pictured as a sum of independent N+N interactions. If the particles are produced independently the value of Φ_{p_T} is equal to zero.

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[1] In the original paper [3] the Φ_{p_T} is called ΔD. Here we follow notation introduced by the NA49 Collaboration, which relates the name of the fluctuation measure to the variable in which fluctuations are studied

3 Formulation of the CF–method

The TF–method can be converted into a method allowing to study event–by–event fluctuations of the "chemical" composition (relative number of different hadronic states) of the collisions (CF–method). In the latter method the basic quantity in which fluctuations are analyzed is the number of particles of a given type produced in a single collision. This number substitutes the transverse momentum of particles used in the TF–method (see Sect. 2).

A formal trick which allows for a direct conversion of the TF–method to the CF–method is based on the substitution of the kinematical variable used for "thermal" fluctuation studies (e.g. p_T as discussed in Sect. 2) by a function defined as:

$$\delta(h^i, h_0) = \begin{cases} 1 & : \quad h^i = h_0 \\ 0 & : \quad h^i \neq h_0 \end{cases} \tag{2}$$

where i is a particle index, h^i is the particle type of the particle i and h_0 is the particle type selected for the fluctuation analysis. Then the variable z takes the form:

$$z_i = \delta(h^i, h_0) - \overline{\delta(h, h_0)},$$

where the $\overline{\delta(h, h_0)}$ is the mean value of $\delta(h^i, h_0)$ calculated for all particles from all events (the inclusive mean). $\overline{\delta(h, h_0)}$ gives the probability that any particle of the event ensamble is of the type h_0. For every event the sum:

$$Z = \sum_{i=1}^{N} z_i,$$

is calculated where the summation runs over all N particles. Finally the "chemical" fluctuation measure, $\Phi(h_0)$, can be calculated as:

$$\Phi(h_0) = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\overline{z^2}}, \tag{3}$$

where $\langle N \rangle$ and $\langle Z^2 \rangle$ are averages (of event–by–event observables) over all events and the second term in the r.h.s. is the square root of the second moment of the inclusive z distribution.

As follows from the construction the important statistical features of the $\Phi(h_0)$ variable are identical to the features of the Φ variable used in the TF–method (e.g. Φ_{p_T}). We list them below.

- For a system which is an independent sum of elementary processes the value of $\Phi(h_0)$ is equal to the value of $\Phi(h_0)$ calculated for a single elementary process independent of the number of superimposed elementary processes and its distribution in the analyzed event sample.
- In the model in which particles are produced independently from each other the value of $\Phi(h_0)$ is equal to zero.

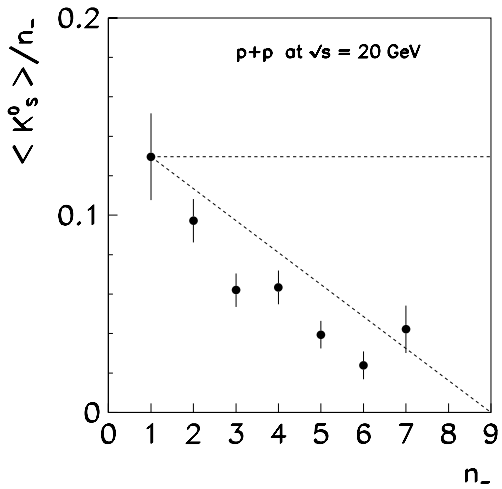

Fig. 1. The dependence of the ratio $\langle K_S^0 \rangle / n_-$ on negative hadron multiplicity, n_-, for p+p interactions at $\sqrt{s} = 20$ GeV

4 Numerical example

We first note that particle production in elementary processes (e.g. p+p interactions) is correlated not only in momentum space [7] but also when the "chemical" composition is considered. In order to illustrate this statement by experimental data [8,9] we show in Fig. 1 the ratio of the mean K_S^0 multiplicity to the multiplicity of negatively charged hadrons, n^-, as a function of n_- for p+p interactions at 200 GeV/c. The data show that the K_S^0 multiplicity decreases significantly with the event multiplicity. A similar correlation can be expected for K^- multiplicity, and in fact, it is observed in string models of p+p interactions [10]. It means that the probability that a negatively charged hadron is K^- meson depends on the multiplicity of negatively charged hadrons in the event. Thus particles are not produced independently and therefore we expect that the value of $\Phi(K^-)$ calculated for negatively charged hadrons for p+p interactions is not equal to zero. In order to make numerical estimation of the effect we use a simple parametrization of hadron production in p+p interactions at 200 GeV/c.

It is assumed that a dependence of $\langle K^- \rangle / n_-$ on n_- is similar to the dependence of $\langle K_S^0 \rangle / n_-$ on n_- shown in Fig. 1. The multiplicity distribution of negatively charged hadrons is calculated using the parametrization from [11]. Further we assume that the multiplicity distribution of K^- mesons for a fixed multiplicity n_- is given by the binominal distribution i.e.:

$$P(n_K; n_-) = \binom{n_-}{n_K} P_K(n_-)^{n_K} (1 - P_K(n_-))^{n_- - n_K}, \tag{4}$$

where n_K is the kaon multiplicity and $P_K(n_-)$ is the probability that a negatively charged hadron is K^-–meson. Following the data presented in Fig. 1 the $P_K(n_-)$ is parametrized as:

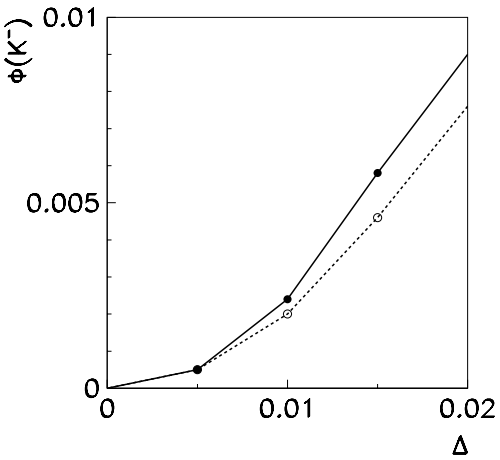

Fig. 2. The dependence of the fluctuation measure $\Phi(K^-)$ on the paremeter Δ (see text) for two values of the parameter $P_K(1)$: 0.13 (solid line) and 0.26 (dashed line)

$$P_K(n_-) = \begin{cases} P_K(1) - (n_- - 1)\cdot\Delta \;:\; P_K(1) > (n_- - 1)\cdot\Delta \\ 0 \;:\; P_K(1) \leq (n_- - 1)\cdot\Delta \end{cases} \quad (5)$$

In order to study the dependence of $\Phi(K^-)$ on the correlation between $P_K(n_-)$ and n_- the parameter Δ is varied between 0 (no correlation) and 0.02 (a correlation as suggested by the data presented in Fig. 1). The parametrizations of $P_K(n_-)$ obtained for these two extreme values of Δ are indicated by dashed lines in Fig. 1 for $P_K(1) = 0.13$.

The dependence of $\Phi(K^-)$ on the parameter Δ is shown in Fig. 2 for two values of the parameter $P_K(1)$: 0.13 (solid line) and 0.26 (dashed line). For $\Delta = 0$ the value of $\Phi(K^-)$ is equal to zero, as expected from the definition of Φ. The value of $\Phi(K^-)$ increases with increasing Δ, i.e. with the increasing correlation between K^- yield and the event multiplicity.

Using the above model ($P_K(1) = 0.13$ and $\Delta = 0.02$) we estimated that the number of events needed to obtain 10% statistical error on $\Phi(K^-)$ is about 10^5.

5 Summary

We proposed a method to study event–by–event fluctuations of the "chemical" composition of the final state of high energy collisions. A simple numerical example of fluctuations of the number of K^-–mesons in the sample of negatively charged hadrons was considerd. The method can be used to study a change of the magnitude of the "chemical" fluctuations when changing the size of the colliding systems (p+p, p+A and A+A) and/or when changing the collision energy.

We expect that the method will allow to determine whether or not chemical equilibration is reached in high energy nucleus–nucleus collisions.

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